

GENERATION OF A MAGNETIC FIELD IN THE SHEAR DEFORMATION REGION OF A CONDUCTING MATERIAL UPON HIGH-VELOCITY PENETRATION

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UDC 533.95

It is shown that when a high-velocity impactor penetrates into a conducting target with a transverse magnetic field, conditions for considerable field amplification are produced in the shear deformation region on the lateral surface of the impactor. Field generation in a conducting medium deformed in shear is considered within the framework of a plane one-dimensional problem of magnetohydrodynamics. The results obtained indicate that along the boundary of the cavity produced by the impactor in the target with a magnetic field, a thin layer with a very high field intensity (about 100 T) is formed. The possibility of explosion of this layer due to the magnetic pressure acting in it is analyzed.

Generation of a strong magnetic field, which is responsible for additional mechanical, thermal, and electromagnetic effects, can strongly affect the course of various physical processes. Among the processes that lead to magnetic-field amplification with manifestation of associated effects is specifically organized motion of a conducting medium with a rather weak field produced beforehand in the medium.

The conditions of motion that ensure field generation follow from the equation describing the magnetic-field evolution in a moving conducting medium [1]:

$$\frac{d}{dt} \left(\frac{\mathbf{B}}{\rho} \right) = \frac{\mathbf{B}}{\rho} \nabla \cdot \mathbf{v} + \frac{1}{\mu_0 \sigma \rho} \Delta \mathbf{B}. \quad (1)$$

Here \mathbf{B} is the magnetic induction vector, \mathbf{v} is the particle velocity vector, ρ is the density of the medium, σ is the electrical conductivity of the medium, and $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the permeability of vacuum.

According to the magnetic field “freezing-in” effect [1], described by the first term on the right side of Eq. (1), in a moving conducting medium, the magnetic flux lines move together with material particles through which they pass. In this case, the quantity B/ρ should vary along each magnetic line in proportion to its local elongation. If we ignore the compressibility of the medium ($\rho = \text{const}$), this corresponds to a simultaneous change in the length of the material fibres, oriented along the magnetic lines, and the magnetic induction B in the material. Thus, amplification of a magnetic field produced in a moving incompressible conducting material should be observed in the material regions whose motion is accompanied by deformations of particles leading to elongation of the magnetic flux lines. Diffusion processes in the material, described by the second term on the right side of Eq. (1), whose rate increases with decrease in the electrical conductivity of the material σ , apparently leads to weakening of magnetic-field generation due to “dispersal” of field nonuniformities at the moment they form.

The conditions of motion necessary for magnetic-field generation arise in a conducting target penetrated by a high-velocity impactor which moves across the magnetic flux lines of the initial field produced in the target [2]. A possible generation mechanism that occurs in the target region just ahead of the penetrating impactor is described in [3]. This mechanism is associated with extremely large shear strains of particles of the target layer adjacent to the impactor head in a direction perpendicular to the direction of penetration. (i.e., in the direction of the field produced in the target).

Bauman Moscow State Technical University, Moscow 107005. Translated from *Prikladnaya Mekhanika i Tekhnicheskaya Fizika*, Vol. 42, No. 3, pp. 15–23, May–June, 2001. Original article submitted December 10, 1999; revision submitted October 3, 2000.

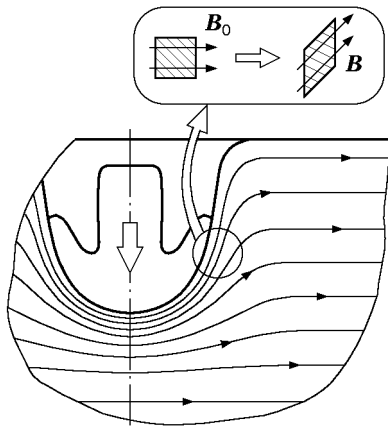


Fig. 1

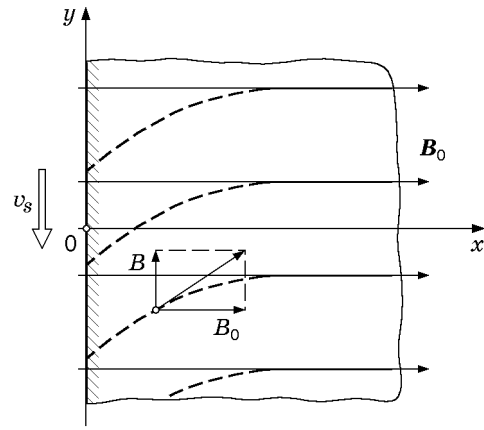


Fig. 2

Fig. 1. Magnetic-field amplification in a conducting target deformed in shear upon penetration of an impactor.

Fig. 2. Calculated diagram of magnetic-field generation in a conducting target under shear deformation.

In the present paper, we consider a magnetic-field generation mechanism that may occur upon high-velocity penetration into a conducting material with a transverse magnetic field. This mechanism is due to large shear strains of the target particles located in the lateral surface layer of the cavity produced by an impactor. Magnetic-field amplification in this target region is due to the fact that the material fibers present in the target, which are originally oriented, as the field, perpendicular to the direction of motion of the impactor, turn in the penetration direction during interaction, and this leads to an increase in their length (Fig. 1). According to the “freezing-in” effect, the magnetic field thus generated in the lateral layer of the cavity also has longitudinal orientation.

A basic understanding of magnetic-field generation in the shear deformation region of a target upon high-velocity penetration can be obtained using the following simplified model of the process within the framework of a plane one-dimensional problem of magnetohydrodynamics.

Since at high velocities of interaction (several kilometers per second), penetration proceeds in a nearly hydrodynamic regime [4], we ignore the strength properties of the target material and use the model of an incompressible viscous liquid. Let us consider a half-space occupied by an incompressible viscous conducting medium in which a uniform magnetic field with induction B_0 normal to the boundary of the half-space is produced (Fig. 2). We direct the x and y coordinate axes into the depth of the medium and along its boundary, respectively, and take the boundary as the origin of the x coordinate. At the initial time, the medium is at rest. In simulating the action of the impactor on the target material adjacent to the lateral surface of the impactor, we assume that the tangential stresses acting on the boundary of the half-space set it in motion along the y axis. In addition, over time v_s , the velocity of the boundary τ increases linearly, reaching the value v_0 , and then remains unchanged: $v_s = v_0 t / \tau$ at $0 \leq t \leq \tau$ and $v_s = v_0$ at $t > \tau$. The possibility that the boundary of the medium moves along the x axis is excluded.

The motion of the medium that arises under the conditions formulated above is obviously characterized by the single velocity vector component v directed along the y axis. In this case, a magnetic field with the induction component B of the same direction should be produced in the medium. The quantities v and B are functions of time t and the space variable x . The magnetic induction vector component in the x direction does not vary, remaining equal to the induction B_0 of the initial field produced in the medium. The interaction of the field B_0 with the induction currents of bulk density $j = (\partial B / \partial x) / \mu_0$ flowing normal to the coordinate plane xy gives rise to volumetric electromagnetic forces directed along the y axis.

In view of the aforesaid, the equation of motion for the material particles acted upon by electromagnetic and viscous forces is written as

$$\rho \frac{\partial v}{\partial t} = \frac{B_0}{\mu_0} \frac{\partial B}{\partial x} + \eta \frac{\partial^2 v}{\partial x^2}, \quad (2)$$

where η is the dynamic viscosity of the medium.

In this case, Eq. (1), which describes the evolution of the magnetic field in the medium, becomes

$$\frac{\partial B}{\partial t} = B_0 \frac{\partial v}{\partial x} + \frac{1}{\mu_0 \sigma} \frac{\partial^2 B}{\partial x^2}. \quad (3)$$

Introducing the dimensionless time $t' = t/\tau$ and the coordinate $x' = x/(c_A \tau)$, where $c_A = B_0/\sqrt{\mu_0 \rho}$ is the Alfvén velocity [1], we write relations (2) and (3) in the form

$$\frac{\partial v'}{\partial t'} = \frac{\partial B'}{\partial x'} + \beta^2 \frac{\partial^2 v'}{\partial x'^2}, \quad \frac{\partial B'}{\partial t'} = \frac{\partial v'}{\partial x'} + \varkappa^2 \frac{\partial^2 B'}{\partial x'^2}. \quad (4)$$

The dimensionless velocity v' and the magnetic induction B' of the field are defined by

$$v' = v/v_0, \quad B' = B/(v_0 \sqrt{\mu_0 \rho}). \quad (5)$$

The solution of system (4) depends on the two nondimensional parameters β and \varkappa :

$$\beta = \frac{1}{B_0} \sqrt{\frac{\mu_0 \eta}{\tau}}, \quad \varkappa = \frac{1}{B_0} \sqrt{\frac{\rho}{\sigma \tau}}. \quad (6)$$

In the case of an inviscid ($\eta = 0$) theoretically conducting ($\sigma \rightarrow \infty$) medium, the parameters β and \varkappa are equal to zero, and system (4) reduces to the following wave equations for v' and B' :

$$\frac{\partial^2 v'}{\partial t'^2} = \frac{\partial^2 v'}{\partial x'^2}, \quad \frac{\partial^2 B'}{\partial t'^2} = \frac{\partial^2 B'}{\partial x'^2}.$$

The solution of these equations is represented as an Alfvén wave running along the x axis [1]. Behind the front of this wave, the quantities v' and B' are equal to unity:

$$v' = B' = 0 \quad \text{for} \quad t' - x' \leq 0, \quad v' = B' = t' - x' \quad \text{for} \quad 0 < t' - x' \leq 1, \\ v' = B' = 1 \quad \text{for} \quad t' - x' > 1.$$

The parameters β and \varkappa characterize the effects of dissipative processes due to viscosity and electric resistance on the propagation of the Alfvén wave and have the following physical meaning. The parameter $\beta = l_v/l_A$ is the ratio of the characteristic thickness of the viscous material layer $l_v = \sqrt{\eta \tau / \rho}$ that is set in motion in time τ to the path of the Alfvén wave $l_A = c_A \tau$ over the same period of time, $\varkappa = l_d/l_A$ is the ratio of the characteristic depth of magnetic-field diffusion $l_d = \sqrt{\tau / (\mu_0 \sigma)}$ in time τ to the quantity l_A .

System (4) with varied values of the parameters β and \varkappa was integrated numerically by a finite-difference method [5]. For the magnetic field on the boundary of the medium $x' = 0$, we used the boundary condition $\partial B'/\partial x' = 0$, which simulates the ideal conductivity of the boundary and ignores field diffusion into the ambient half-space, in which, according to the formulation of the problem, there is the impactor material.

In determining the values of the parameters β and \varkappa that correspond to the conditions of high-velocity interaction between the impactor and target, it was assumed that the target is made of a metal with electrical conductivity typical of copper or aluminum, the induction of the initial magnetic field B_0 is about 10 T, and the time of acceleration of the boundary τ is in the microsecond range. The dynamic viscosity of the target material, according to the data of [6] for high-rate deformation of metals, was set equal to 10^3 – 10^4 Pa · sec. As estimates showed, the values of both parameters that are of interest for the solution of the present problem are in the range of 1 to 10.

In analysis of the results obtained, it is of interest to consider the evolution of the magnetic field in the medium. Under the conditions formulated above, the induction of the generated field B' is a symmetric function of the parameters β and \varkappa : $B'(\beta, \varkappa) = B'(\varkappa, \beta)$. After elimination of the particle velocity v' , system (4) reduces to the differential relation [7]

$$\frac{\partial^2 B'}{\partial t'^2} - \frac{\partial^2 B'}{\partial x'^2} = (\beta^2 + \varkappa^2) \frac{\partial^3 B'}{\partial t' \partial x'^2} - \beta^2 \varkappa^2 \frac{\partial^4 B'}{\partial x'^4},$$

which remains unchanged when β is replaced by \varkappa and back. Thus, in studying the effect of these parameters on the variation of the quantity B' , it suffices to assume a pair of values for them without exactly specifying to which of the parameters they belong. In view of this, the conditions of magnetic-field generation are characterized only by a pair of numbers (enclosed in parentheses in Figs. 3 and 4), implying two possible combinations of the determining parameters.

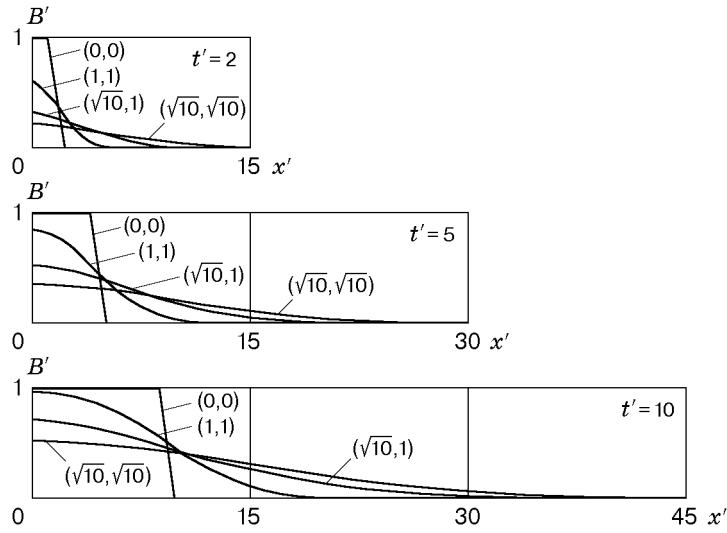


Fig. 3. Distribution of the generated magnetic field induction over the thickness of a conducting target under shear deformation for various times.

From the symmetry of the function B' about the parameters β and α it follows that the dissipative processes due to viscosity and electric resistance have identical effects on the magnetic-field evolution in the medium. Spatial distributions of the induction of the magnetic field generated in the medium for various values β and α are given in Fig. 3 for various times. An increase in the determining parameters leads to “smearing” of the Alfvén wave front. As a result, the extent of the perturbed region of the medium increases and the growth rate of the field intensity in it decreases. In this case, the ultimate value to which the quantity B' tends remains unchanged and equal to unity. Activation of the dissipative processes results in an increase in the time of attainment of this value and earlier occurrence of the field induced in the medium.

For the induction of the magnetic field generated on the boundary of the medium (at $x' = 0$) in the problem considered, it is possible to obtain an analytical solution that can be used to check the accuracy of numerical calculations. Performing an integral Laplace transform [8] of system (4) and integration of the resulting ordinary differential linear second-order equations leads to the following expressions for the images of the magnetic induction

$$B^*(x', p) = \int_0^{\infty} B'(x', t') \exp(-pt') dt' \text{ and the particle velocity } v^*(x', p) = \int_0^{\infty} v'(x', t') \exp(-pt') dt':$$

$$B^* = A_1 \exp(\lambda_1 x') + A_2 \exp(\lambda_2 x'), \quad v^* = A_1 \frac{\lambda_1}{p - \beta^2 \lambda_1^2} \exp(\lambda_1 x') + A_2 \frac{\lambda_2}{p - \beta^2 \lambda_2^2} \exp(\lambda_2 x').$$

Here A_1 and A_2 are the constants of integration and λ_1 and λ_2 are two negative roots of the biquadratic characteristic equation

$$\lambda^4 - \frac{1 + p(\alpha^2 + \beta^2)}{\alpha^2 \beta^2} \lambda^2 + \frac{p^2}{\alpha^2 \beta^2} = 0,$$

which are chosen from the condition of boundedness of the solution as $x' \rightarrow \infty$ and have the following forms:

$$\lambda_1 = -\frac{\sqrt{1 + p(\alpha^2 + \beta^2)} - \sqrt{(1 + p(\alpha^2 + \beta^2))^2 - 4p^2 \alpha^2 \beta^2}}{\sqrt{2} \alpha \beta},$$

$$\lambda_2 = -\frac{\sqrt{1 + p(\alpha^2 + \beta^2)} + \sqrt{(1 + p(\alpha^2 + \beta^2))^2 - 4p^2 \alpha^2 \beta^2}}{\sqrt{2} \alpha \beta}.$$

Determining the constants of integration A_1 and A_2 from the boundary conditions

$$\left. \frac{\partial B^*}{\partial x'} \right|_{x'=0} = 0, \quad v^*(0, p) = \begin{cases} 1/p^2, & 0 \leq t' \leq 1, \\ (1 - \exp(-p))/p^2, & t' > 1 \end{cases}$$

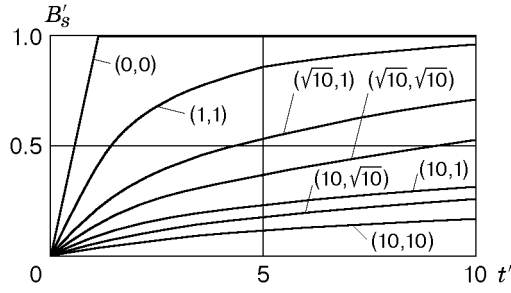


Fig. 4. Magnetic induction of the generated field on the boundary of a conducting target under shear deformation versus time.

written with allowance for the Laplace transform, we write the image of the magnetic-induction function on the boundary of the medium in the form

$$B_s^*(p) = B^*(0, p) = \begin{cases} \frac{1}{p^2 \sqrt{1 + p(\alpha + \beta)^2}}, & 0 \leq t' \leq 1, \\ \frac{1 - \exp(-p)}{p^2 \sqrt{1 + p(\alpha + \beta)^2}}, & t' > 1. \end{cases}$$

Inversion of these relations [9] gives the following law of variation of the generated magnetic field B'_s on the boundary of the medium:

$$B'_s = \begin{cases} t' \operatorname{erf}\left(\frac{\sqrt{t'}}{\alpha + \beta}\right) + \frac{\alpha + \beta}{\sqrt{\pi}} \sqrt{t'} \exp\left(-\frac{t'}{(\alpha + \beta)^2}\right) - \frac{(\alpha + \beta)^2}{2} \operatorname{erf}\left(\frac{\sqrt{t'}}{\alpha + \beta}\right), & 0 \leq t' \leq 1, \\ t' \operatorname{erf}\left(\frac{\sqrt{t'}}{\alpha + \beta}\right) + \frac{\alpha + \beta}{\sqrt{\pi}} \sqrt{t'} \exp\left(-\frac{t'}{(\alpha + \beta)^2}\right) - \frac{(\alpha + \beta)^2}{2} \operatorname{erf}\left(\frac{\sqrt{t'}}{\alpha + \beta}\right) - (t' - 1) \operatorname{erf}\left(\frac{\sqrt{t' - 1}}{\alpha + \beta}\right) \\ - \frac{\alpha + \beta}{\sqrt{\pi}} \sqrt{t' - 1} \exp\left(-\frac{t' - 1}{(\alpha + \beta)^2}\right) + \frac{(\alpha + \beta)^2}{2} \operatorname{erf}\left(\frac{\sqrt{t' - 1}}{\alpha + \beta}\right), & t' > 1. \end{cases}$$

Here $\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-\xi^2) d\xi$ is an error integral [8]. Studying the behavior of the image $B_s^*(p)$ for $t' > 1$ and the transformation parameter tending to zero, it is easy to establish the asymptotic behavior of the quantity B'_s as $t' \rightarrow \infty$:

$$B'_s \approx 1 - O\left(\frac{\exp(-t'/(\alpha + \beta)^2)}{\sqrt{t'}}\right).$$

Figure 4 shows a curve of $B'_s(t')$. It is evident that with increase in the parameters β and α , the growth rate of the magnetic field induction B'_s on the boundary of the medium decreases.

As follows from the results for the model considered, irrespective of the intensity of the initial magnetic field, shear deformation of the medium leads to generation in it of a field with ultimate induction $B = v_0 \sqrt{\mu_0 \rho}$, according to the second relation of (5). For an impactor penetration velocity v_0 of several kilometers per second, the generated field induction should reach a value of about 100 T. The time of generation and the variation in the thickness of the material layer in which there was field amplification depend on the induction of the initially generated field B_0 in relation (6), which defines the parameters β and α . For B_0 of about 10 T under high-velocity penetration conditions ($\tau \sim 1 \mu\text{sec}$), the time of field amplification to 100 T in the shear deformation region of the target can be about 10 μsec . The dimension of the generation region corresponding to this time is several millimeters.

Thus, when a high-velocity impactor penetrates into a conducting target with a transverse magnetic field, a thin layer with a very strong field adjacent to the lateral surface of the impactor should form in the target material. In reality, the formation of this layer is obviously related not only to its shear deformation but also to the generation of a field in the target region just ahead of the impactor [3] due to extension of particles of this region in the direction of the field produced in the target. Elongating in the transverse direction, the particles on the impactor trajectory

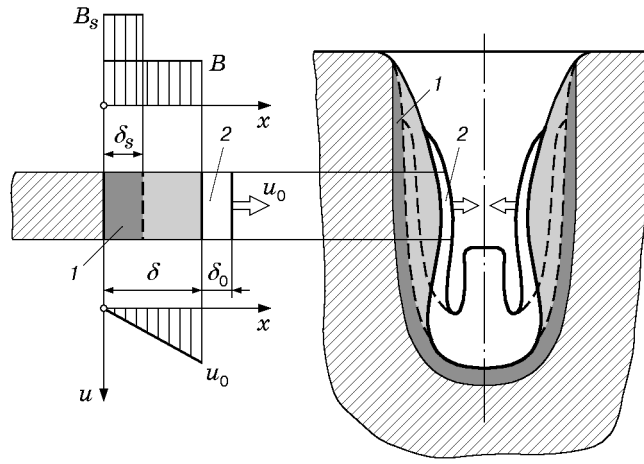


Fig. 5. Calculated diagram of collapse of a cavity produced by an impactor in a target with a magnetic field: 1) target material layer with high magnetic-field intensity; 2) worn-out material of the impactor.

are displaced aside and are brought in the material layer on the lateral surface of the cavity, thus maintaining the high field intensity acquired as a result of deformation.

The formation of a “magnetic” layer along the boundary of the cavity with field intensity estimated at about several hundred Tesla can result in strong thermal and mechanical effects, capable of influencing the penetration process. Explosion of the “magnetic” layer with collapse of the cavity produced by the impactor appears to be most probable.

According to estimates in [3], during “pumping” of a magnetic field to 100 T in a metal layer 1 mm thick, the resulting induction currents ensure a heating rate of this layer as high as 1000 deg/ μ sec. We assume that the “magnetic” layer material at the surface of the cavity is completely softened upon Joule heat release. Then, the magnetic pressure in this layer $p_m = B^2/(2\mu_0)$ [10] of about 10 GPa for $B \sim 100$ T, which is comparable to the pressure of explosion of a blasting charge [11], should lead to expansion of the cavity surface material toward its center. In high-velocity penetration there is wear of the impactor due to spread of its material [4]. Therefore, in the computational model, we take into account that at the cavity surface, the expanding “magnetic” sheet of the target material is in contact with the worn-out layer of the impactor material and accelerates this layer during expansion (Fig. 5). The acceleration velocity of the residual layer of the impactor, and hence, the collapse velocity of the cavity can be estimated using the Gurney approach [11], which is employed to determine the acceleration of explosive detonation products.

As above, using a planar interaction diagram, we assume that during magnetic-field generation upon penetration of an impactor into the target material, a “magnetic” layer of thickness δ_s with a longitudinal field was formed along the lateral surface of the cavity. For simplicity, the magnetic induction B_s is considered constant over the entire thickness of the layer. Entering the vapor state under intense heat, the material of the “magnetic” layer can expand freely, pushing forward the worn-out layer of the impactor material of unchanged thickness δ_0 with no magnetic field. The transverse distribution (along the x axis) of the velocity u of various particles of the “magnetic” sheet during its expansion is assumed to be linear with zero value on the boundary with the rigid part of the target and with the value of u_0 corresponding to the velocity of the residual layer of the impactor (Fig. 5):

$$u = u_0 x / \delta \quad (7)$$

(δ is the current value of the “magnetic” layer thickness).

Determining the collapse velocity of the cavity walls, we ignore the pressure in the “magnetic” sheet due to thermal factors and assume that the collapse is due only to magnetic pressure. In this case, the equation of balance of various types of energy per unit area of the lateral surface of the cavity in the process of collapse can be written as

$$W_{ms} = W_m + E_k + E_{k0}, \quad (8)$$

where W_{ms} and W_m are, respectively, the initial and current energies of the magnetic field in the “magnetic” layer and E_k and E_{k0} are the kinetic energies of the target and impactor materials, respectively.

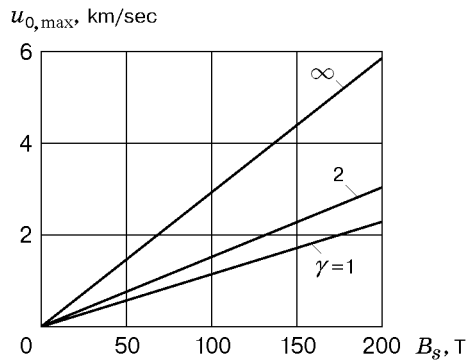


Fig. 6. Ultimate acceleration velocity of the surface layer of a cavity in an aluminum target versus magnetic-field intensity in the layer.

Under the assumption of no magnetic flux losses during expansion of the “magnetic” sheet, the entire initial store of magnetic energy W_{ms} should convert to the kinetic energy of acceleration of the surface layers of the cavity. Thus, the maximum velocity of collapse of the cavity is reached for unbounded expansion of the “magnetic” sheet ($\delta \rightarrow \infty$), which ensures complete exhaustion of magnetic energy in it ($W_m \rightarrow 0$). In view of this, from the energy conservation law (8) into which we substitute the expressions $W_{ms} = B_s^2 \delta_s / (2\mu_0)$ and $E_{k0} = \rho_0 \delta_0 u_0^2 / 2$ (ρ_0 is the density of the impactor material) and the value of $E_k = \rho \delta_s u_0^2 / 6$ (ρ is the density of the target material) determined with allowance for (7), we obtain the following expression for the ultimate velocity of cavity collapse:

$$u_{0,\max} = \frac{B_s}{\sqrt{\mu_0 \rho}} \sqrt{\frac{3\gamma}{3 + \gamma}}.$$

Here $\gamma = \rho \delta_s / (\rho_0 \delta_0)$ is the nondimensional load factor which describes the ratio of the masses of the expanding “magnetic” layer of the target and the accelerated (inert) layer of the impactor material.

Figure 6 shows dependences of the final velocity of “magnetic” acceleration on the magnetic-field intensity B_s in the surface layer of the cavity for various load factors γ for penetration into an aluminum target ($\rho = 2700 \text{ kg/m}^3$). These dependences support the hypothesis on the possibility of collapse of the cavity. Even ignoring the contribution of intense Joule heating of the surface layer of the cavity, the acceleration velocity of the layer for the field intensities ensured by high-velocity penetration (about 100 T) reaches several kilometers per second and corresponds to the velocities of bodies accelerated by condensed explosives [11]. The powerful pulsed action on the penetrating impactor due to explosion of the surface layer of the cavity can lead to failure of the impactor and decrease in its penetrability.

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